READING GROUP: ALGEBRAIC K-THEORY
SUMMER TERM 2026

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The plan for this reading group is to provide a modern introduction to algebraic K-theory using the language of ∞-categories and higher algebra. Our main source will be the lecture notes by Hilman-McCandless [HMC24] from the lecture course taught in the summer term 2024. You can find these lecture notes here. Another good source is the lecture notes by Winges, which you can find here.

We will begin by recalling basic concepts from higher category theory, such as localizations, presentability, stability, $\mathbb{E}_\infty$-groups and -monoids, and symmetric monoidal ∞-categories. After that we can get started with the actual contents; that is, different forms of (algebraic) K-theory. In particular, we will focus on the K-theory of stable ∞-categories through the group completion construction, the Q-construction, and the S-construction. This can, of course, be applied to $\mathrm{Perf}(X)$ of a scheme X or $\mathrm{Perf}(R)$ of a ring R to recover the classical objects of interest.

After this, the participants will hopefully have a basic understanding of the modern approach to algebraic K-theory, and can move on to investigate more advanced topics in the field. As a teaser, there is a possibility of a follow-up reading group on trace methods $(\mathrm{THH}, \mathrm{TC}, \mathrm{TC}^-, \ldots)$ and cyclotomic spectra in the winter term 2026/2027.

Prerequisites

Although we will recall some higher category theory, it is recommended that you have been exposed to this before, e.g. through the higher category theory reading group from the winter term 2025/2026. Other than this, you should be able to get by with just some basic knowledge on algebra, homotopy theory, and ordinary category theory.

Schedule

The reading group will meet on tba at tba in SemR tba.

No.	Date	Topic	Speaker
0.	x	Preliminary meeting	Organizer
1.	x	Localizations, semi-additivity, and additivity	x
2.	x	Symmetric monoidal ∞-categories and $\mathbb{E}_\infty$-algebras	x
3.	x	Group-completion K-theory	x
4.	x	Cyclic invariance and the plus construction	x
5.	x	The Q-construction and Verdier sequences	x
6.	x	Waldhausen's additivity theorem and the universal property of $\mathcal{K}$	x
7.	x	Verdier localization	x
8.	x	Karoubi localization	x
9.	x	Waldhausen categories and the S-construction	x
10.	x	The +=S theorem and dévissage arguments	x
11.	x	The theorem of the t-heart	x
12.	x	The Blumberg–Mandell regularity localization sequence	x
13.	x	Burklund–Levy unipotent dévissage	x

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Syllabus

Talk 0: Preliminary meeting. During this meeting, we will provide an overview of the topic and assign talks. If you are interested in giving a talk, please attend the meeting. Any remaining talks will be available afterward, and you may contact me directly if you would like to present.

Talk 1: Localizations, semi-additivity, and additivity. Cover subsections 2.1–2.3 (roughly pages 13-21). You can be quick on presentable categories, but make sure to focus on the subsection for (semi-)additivity and $\mathbb{E}_\infty$-groups/monoids. Recall the usual group completion from basic algebra and compare it to our definition.

Talk 2: Symmetric monoidal ∞-categories and $\mathbb{E}_\infty$-algebras. Cover subsections 2.4-2.6 (pages 22-32). You can be very brief on stable ∞-categories and focus on subsections 2.5-2.6. Do your best to explain symmetric monoidal ∞-categories and $\mathbb{E}_\infty$-algebras in light of the fact that we will not talk about ∞-operads. This talk certainly requires a solid prior knowledge of the material, so the speaker should preferably be familiar with Lurie's Higher Algebra [Lur17].

Talk 3: Group-completion K-theory. Cover subsection 3.1 (roughly pages 33-38). This has relatively little content, so you can choose to flesh out the details and/or add examples from external sources. Room for more examples and external sources.

Talk 4: Cyclic invariance and the plus construction. Cover subsections 3.2-3.3. (pages 39-51), but you may focus on 3.2. You may want to consult other more classical sources (e.g. [Wei13]) for more intuition on the plus construction.

Talk 5: The Q-construction and Verdier sequences. Cover subsections 4.1-4.2 (roughly pages 52-64). This has a lot of material, so you will again need to pick and choose. You can definitely leave out some of the technical exercises. You may want to consult [Wei13] for intuition on the construction.

Talk 6: Waldhausen's additivity theorem and the universal property of $\mathcal{K}$. Cover subsection 4.3 (roughly pages 65-76). Try to give a complete proof of the theorem, modulo some of the intricate details regarding p-cocartesian morphisms. Focus on the universal property of K-theory.

Talk 7: Verdier localization. Cover subsection 4.4 (roughly pages 77-82). As this subsection is not very long, you can either choose to come up with external examples or go deeper into the theory (cf. [HLS23] for background and references). Another option is to already get started on Karoubi localization to lessen the workload of the next speaker; please be in contact with the next person in this case.

Talk 8: Karoubi localization. Cover subsection 4.5 (pages 83-98). This subsection is very long, so you will surely not be able to cover everything. You may skip some proofs and/or agree to have the previous speaker start on the subsection. Make sure to talk about nonconnective K-theory.

Talk 9: Waldhausen categories and the S-construction. Cover subsections 5.1-5.2 (pages 99-106). If you have time, you can sketch the proof of Theorem 5.2.16.

Talk 10: The +=S theorem and dévissage arguments. Cover subsections 5.3-5.4 (roughly pages 107-117). This is very technical, so you will need to decide how much of it to present and in what amount of detail.

Talk 11: The theorem of the t-heart. Cover subsections 5.5-5.6 (roughly pages 117-129). Recall the basics of t-structures on stable ∞-categories and go from there. If you have time, define weight structures and state the theorem of the weight heart (Theorem 5.1.5).

Talk 12: The Blumberg–Mandell regularity localization sequence. Cover subsections 5.7-5.8 (roughly pages 130-138). Make sure to discuss the examples regarding real and complex K-theory in detail.

Talk 13: Burklund–Levy unipotent dévissage. Cover subsection 5.9 (roughly pages 139-144). This subsection is very short, so you will want to consult the original source [BL21a]. For context, you may also want to look up the (incomplete) script [BL21b], which you can find here.

References

[BL21a] Robert Burklund and Ishan Levy, *On the k-theory of regular coconnective rings*, 2021. arXiv: 2112.14723 [math.KT].

[BL21b] ———, *Some aspects of noncommutative geometry*, Incomplete, 2021.

[HLS23] Fabian Hebestreit, Andrea Lachmann, and Wolfgang Steimle, *The localisation theorem for the* K-*theory of stable* ∞-*categories*, 2023. arXiv: 2205.06104 [math.KT].

[HMC24] Kaif Hilman and Jonas McCandless, *Introduction to algebraic k-theory*, 2024.

[Lur17] Jacob Lurie, *Higher algebra*, https://www.math.ias.edu/~lurie/, 2017.

[Wei13] C.A. Weibel, *The K-book: An Introduction to Algebraic K-theory* (Graduate Studies in Mathematics). American Mathematical Society, 2013, ISBN: 9780821891322.